Bridges Math 122A
Fall 2014

Name _____ _____
(GIVEN) (LAST)

## EXPONENTIAL FUNCTIONS [1.2]

**EX 1** As $c \to \infty$, $\left(1+\frac{1}{c}\right)^c \to$ _____.

To do this, use your calculator to graph $y = \left(1+\frac{1}{x}\right)^x$ with $x \in [0, 100]$ and $y \in [0, 3]$.
As x approaches 100, what do you get for y?
Now extend the $x$ values to be $x \in [0, 10000]$ and let $x$ approach 10000.
What is $y$ approaching?

## FORMS OF AN EXPONENTION FUNCTION

**[1]** $P = P_0 a^t$ where $P_0$ is the initial quantity
and $a$ is the factor by which $P$ changes when $t$ increases by 1.

**EX 2** How much money results from $P_0$ dollars being invested at 5% annual interest for 3 years?

| | |
|---|---|
| 1st year | |
| 2nd year | |
| 3rd year | |

Write the function for the amount of money that results from $P_0$ dollars being invested at 5% annual interest for $t$ years: _____

Therefore, the exponential function can be written as

**[2]** $P = P_0(1+r)^t$ where $r$ is the "annual" growth rate.
and $a =$ _____

**EX 3** The water in my ferret's container holds 1 L. Write a formula for the quantity $Q$ remaining after $d$ days if:
(a) he drinks 140 mL each day (b) he drinks 12% each day.

If a quantity $P_0$ is compounded $n$ times per time-frame, then

**[3]** $P = P_0\left(1 + \frac{r}{n}\right)^{nt}$

NOTE: IF $t$ IS IN YEARS, THEN $n$ IS THE # OF COMPOUNDING PERIODS PER YEAR.
IF $t$ IS IN MONTHS, THEN $n$ IS THE # OF COMPOUNDING PERIODS PER MONTH.
IF $t$ IS IN MILLENNIA, THEN $n$ IS THE # OF COMPOUNDING PERIODS PER MILLENNIUM.

$n$ can be represented as $rc$ where $c$ is a real number. Let $r = k$ and let $n = kc$ in equation [3].

If a quantity $P_0$ is "compounded" continuously, then $n \to \infty$ and $c \to \infty$.
Our formula [3] becomes

**[4]** _____ where $k$ is the continuous growth rate

**EX 4** Find a formula relating $r$ and $k$.

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**THE GENERAL FORM FOR AN EXPONENTIAL FUNCTION IS** $\boldsymbol{P = P_0 a^t}$ BUT
the independent variable does not always have to be $t$
the dependent variable does not always have to be $P$
the constants do not always have to be $P_0$ and $a$.

Other typical equations are $f(x) = ab^x$ or $Q = Q_0 a^t$

**EX 5** Let's go back to $P = P_0 a^t$ to answer these questions. Assume $P_0 > 0$ and $t > 0$.

(a) If $a > 1$, then the function is _____(increasing or decreasing) and _____(concave up or concave down).
This is called exponential _____.

(b) If $0 < a < 1$, then the function is _____(increasing or decreasing) and _____(concave up or concave down).
This is called exponential _____.

(c) What happens to the graph if $a = 1$? _____

(d) What happens to the graph if $a < 0$? Try graphing $y = (-2)^x$. _____

(e) What is the equation of the horizontal asymptote for $P = P_0 a^t$. _____

**EX 6** Let $f(g) = 4(1.024)^q$ where $q$ is in months.

(a) The initial amount is _____.

(b) The base is _____.

(c) Find the monthly growth rate?

(d) Calculate the yearly growth rate.

**EX 7** Given below is a chart giving the population of a city beginning in 1980.

| Year | 1980 | 1990 | 2000 | 2010 |
|---|---|---|---|---|
| Year $t$ (1980=0) | 0 | 10 | 20 | 30 |
| Popu $P$ in Millions | 5 | 10 | 20 | ? |

Using the equation $P = P_0 a^t$,

(a) $P_0 =$ _____

(b) Calculate the value of $a$.

(c) Give a formula for the data:

**EX 8** **TWO MORE FORMULAS:**

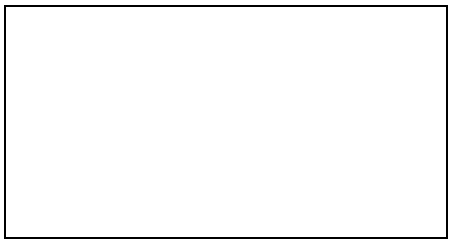
**HALF LIFE:** $Q = Q_0(0.5)^{t/H}$
where $H$ is the half-life.

**DOUBLING TIME:** $Q = Q_0(2)^{t/D}$
where $D$ is the doubling-time.

**EX 9** **SATURATED GROWTH**

The quantity, $Q$, of a drug in a patient's body at time $t$ is represented for positive constants $S$ and $k$ by the function $Q = S(1 - e^{-kt})$.

(a) Sketch a graph of $Q$ if we know that the horizontal asymptote is 9.



What is the equation of this asymptote?

(b) As $t \to \infty$, $e^{-kt} \to$ _____.

As $t \to \infty$, $Q \to$ _____.

**EX 10** Find the equation of an exponential function containing the points $\left(-1, \frac{4}{3}\right)$ and $\left(2, \frac{1}{6}\right)$.

**EX 11** When the Olympic Games were held outside Mexico City in 1968, there was much discussion about the effect the high altitude (7340 feet) would have on the athletes. Assuming air pressure decays exponentially by 0.4% every 100 feet, by what percentage is air pressure reduced by moving from sea level to Mexico City?